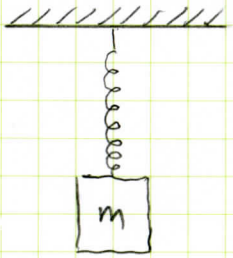
TMS PR 3-2

A MASS OSCILLATING IN A RESISTING MEDIUM HAS ITS AMPLITUDE DECREASE TO $\frac{1}{2}A_0$ IN 10s. FIND

a) β AND

b) ν_s (COMPARE TO ν_0)



$m = 100\,g$
$k = 10^4\,g/cm$
$A_0 = 3\,cm$
$v_0 = 0$

SINCE IT'S OSCILLATING, ASSUME UNDERDAMPED

$$\ddot{x} + 2\beta\dot{x} + \omega_N = 0$$

$$\Rightarrow \quad x(t) = A_0 e^{-\beta t}\cos(\omega_s t - \delta)$$

a) APPLY THE DECREMENT OF MOTION

$$\frac{A(t_0+10_s)}{A(t_0)} = \frac{1}{2} = \frac{A_0 e^{-\beta(t_0+10s)}}{A_0 e^{-\beta(10s)}} = e^{-\beta(10s)}$$

$$\ln 2 = \beta(10s)$$

$$\beta = \frac{\ln 2}{10} = \boxed{0.0693\,s^{-1} = \beta}$$

b) FIND $\omega_N = \sqrt{\frac{k}{m}} = \sqrt{\frac{10^4}{100}} = 10\,s^{-1}$

THEN

$$\nu_s = \frac{\omega_s}{2\pi} = \frac{1}{2\pi}\sqrt{\omega_N^2 - \beta^2} = \frac{\omega_N}{2\pi}\left(1 - \frac{\beta^2}{\omega_N^2}\right)^{1/2}$$

SINCE $\beta \ll \omega_N$, EXPAND

$$\nu_s = \frac{\omega_N}{2\pi}\left(1 - \frac{\beta^2}{2\omega_N^2} + \ldots\right)$$

$$\nu_s = \nu_0\left(1 - \frac{0.0693^2}{100}\right)$$

$$= \nu_0\left(1 - 4.80\times10^{-5}\right)$$

$$\boxed{\nu_s = 0.99995\,\nu_0 = 1.59\,Hz} \Rightarrow \text{Light Damping}$$